$$\Omega_r(\alpha_s) = \xi\frac{(\alpha_2-\alpha_1)^2}{4} - \frac{(v-v_g)^2}{4\lambda};\ \Omega_i(\alpha_s) = \alpha_0(v-v_f) + \varkappa\frac{(v-v_g)^2}{4\lambda},\ v_f = u + \varepsilon\alpha_0.$$

Equation (5) for the second mode yields a satisfactory quantitative approximation. The constants in Eq. (4) for Re = 3000 have the values

$$\alpha_1 \approx 0.186;\quad \alpha_2 \approx 0.243;\quad \xi \approx 4.5;\quad \varepsilon \approx -0.515;\quad u \approx 1.02.$$

LITERATURE CITED

1. H. Schlichting, Boundary Layer Theory, McGraw-Hill (1966).
2. K. Stewartson, and J. T. Stuart, "A nonlinear instability theory for a wave system in plane Poiseuille flow," J. Fluid Mech., 48, Part 3 (1971).
3. A. S. Dryzhov, "Stability of a compressible boundary layer relative to a localized disturbance," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1973).

CALCULATION OF THE INTERACTION OF A TURBULENT BOUNDARY LAYER WITH AN EXTERNAL SUPERSONIC FLOW ON THE CONCAVE CORNER AND ON THE SPHERICAL INTAKE PART OF A BODY

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INTRODUCTION

An integrated method of calculating turbulent flow on two-dimensional and axisymmetrical bodies in separation and attached boundary layer zones arising in the neighborhood of a concave corner and on a spherical intake part of a body is proposed. The method allows us to calculate pressure distribution, displacement thickness, and momentum thickness within the region in which the boundary layer interacts with an external ideal flow. The phenomenon of the interaction between a viscous and nearly inviscous flow is widespread. It is observed when a concave corner is streamlined, as a pressure shock impinges on a boundary layer, in the case of flow in the neighborhood of the spherical intake part of an axisymmetrical body, and in many other cases. The distinctive features of this phenomenon when two-dimensional and axisymmetrical bodies are streamlined has been theoretically investigated in [1-4]. Separated flows due to a pressure shock or an obstacle have been studied in [1-3], while [4] determined the base pressure behind the spherical intake part of a body. Theoretical investigations for the case of "free" separated flows in which the separation point and the attached boundary layer were not fixed, for example, on a plate with long wedge attached to it, have been carried out within the context of boundary-layer theory using integrated methods. In the current article, an integrated method of calculating flows in a base region [5] is used to calculate "free" separated flows in the neighborhood of a concave corner and on a spherical intake part of a body with a base support. The results of the calculations are compared to experimental data.

§1. Let us consider the following approximate flow scheme in the separation zones of a boundary layer in front of a wedge (flap) in the form of a scheme for the ordinary interaction of a turbulent boundary layer with an external ideal flow (Fig. 1). The interaction region is within the separation zone 1-4 and the attached zone 5-8.

In the separation zone, we distinguish gradient flow 1-3 and constant-pressure flow 3-4; S_1S_2 is the constant flow rate line, where S_1 and S_2 are critical points. The calculation of the interaction of viscous layers

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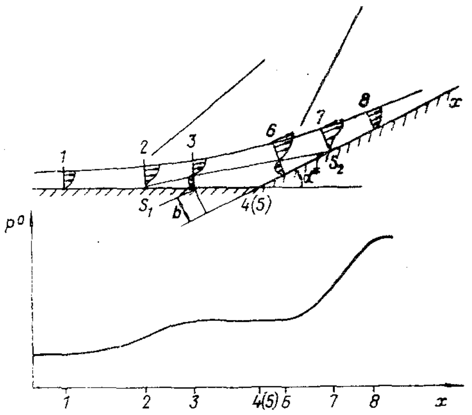

Fig. 1

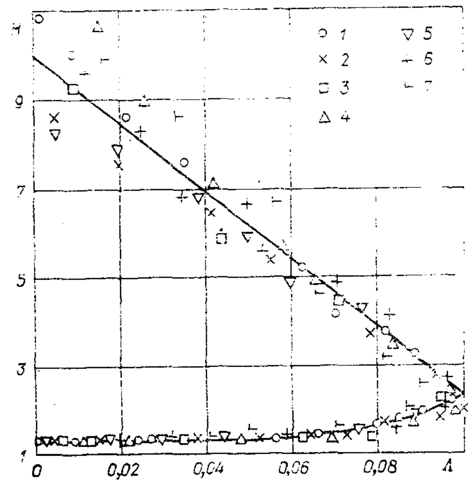

Fig. 2

with an external ideal supersonic flow will be carried out within the framework of boundary-layer equations. According to [5], the following system of equations may be written for the interaction region:

$$d\delta^*/dx = F_0(\mathrm{M},\ \delta^*,\ \theta^{**});\quad (1.1)$$
$$d\theta^{**}/dx = F_1(\mathrm{M},\ \delta^*,\ \theta^{**});$$
$$d\mathrm{M}/dx = F_3(\mathrm{M},\ \delta^*,\ \theta^{**}),$$

where

$$F_0 = \mathrm{tg}\,\beta + D,\ F_1 = \Gamma^2\xi - F_2\frac{\theta^{**}}{\mathrm{M}}(H+2) - \frac{\varepsilon\theta^{**}}{r}\frac{dr}{dx},\quad (1.2)$$
$$F_2 = -\frac{A^*\mathrm{M}\Lambda^2}{T^0 H\delta^{**}},\ F_3 = \mathrm{M}\,\frac{p_1}{p_{01}}\frac{a_{01}}{a_1},\ F_3' = \frac{dF_3}{d\mathrm{M}},$$
$$D = \frac{\delta^*}{F_3}\left(\frac{1}{h^*} - 1\right)F_2F_3',\ \xi = \frac{A^*}{T^0}\left(\frac{I_1}{I_{01}}\right)^{0.5\vartheta},\ \theta^{**} = \delta^{**}$$
$$\times\left(\frac{I_1}{I_{01}}\right)^{0.5}\frac{\rho_1}{\rho_{01}},\ T_0 = 0.5I_w/I_{01} + 0.22\mathrm{Pr}^{1/3} + \left(0.5 - 0.22\mathrm{Pr}^{1/3}\right)I_1/I_{01},$$
$$h^* = \frac{HI_{01}/I_1 + 1/2\,(\varkappa - 1)\,\mathrm{M}^2}{H/H^* + 1/2\,(\varkappa - 1)\,\mathrm{M}^2\,(H+1)}.$$

We note that Eqs. (1.1) were obtained using the Coles–Krokko transformation, by means of which a compressible turbulent layer is transformed into an incompressible boundary layer. We obtain a relation between the parameters λ^2 and h of a compressible boundary layer and the parameters Λ^2 and H of an incompressible layer using previous [5] equations,

$$h = H\frac{I_{01}}{I_1} + \left(\frac{I_{01}}{I_1} - 1\right);\quad (1.3)$$

$$\frac{\lambda^2}{\Lambda^2} = \frac{A^*}{T^0}\frac{h}{H}\frac{I_1}{I_{01}}.\quad (1.4)$$

Equations between the parameters of an incompressible turbulent layer H, H^*, Γ and Λ occurring in the system of equations (1.1) for the zone of an attached boundary layer 1-2 (cf. Fig. 1) are assumed to be known and are selected in the form of dependences $H = H(\Lambda)$, $\Gamma = \Gamma(H)$, and $H^* = H^*(H)$, as presented in [5]. The equations between these parameters will be defined in the zone of circulatory flow 2-3. For this purpose we will jointly solve the first two equations of the system (1.1) and calculate the parameters δ^* and δ^{**} in the separation zone of a two-dimensional boundary layer (the pressure distribution in the interaction region and the variables δ_1^*, δ_1^{**}, $(d\delta^*/dx)_1 = \tan\beta_1$, and M_1 in the initial section 1 are taken from experimental studies [6, 7]). The parameter Γ vanishes in the zone of circulatory flow [5]. In order to take into account undermixing ($D \neq 0$) we will use the method of successive approximations, which consists in first setting $D = 0$ and calculating from the first equation of Eqs. (1.1) the distribution of the parameter $\delta^*(x)$ and from the second equation, that of $\theta^{**}(x)$, the parameter β being found from the Prandtl–Meyer equation. Equations between the parameters H, h, $\delta^{**} = \delta^*/h$, and θ^{**} are used in the course of the calculations.

We find the distribution of H(x) and Λ(x) as we carry out calculations, taking into account Eqs. (1.3) and (1.4). We further compute the distribution of the parameter characterizing undermixing D(x). For

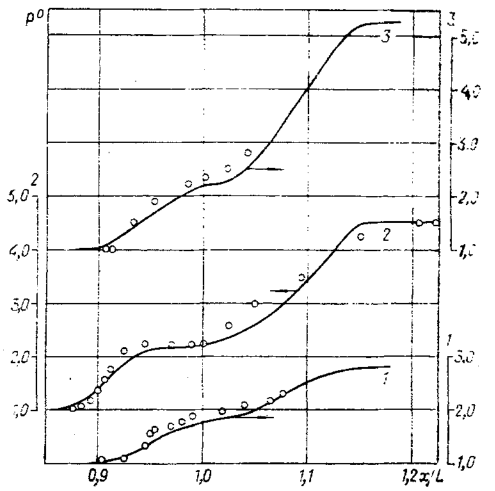

Fig. 3

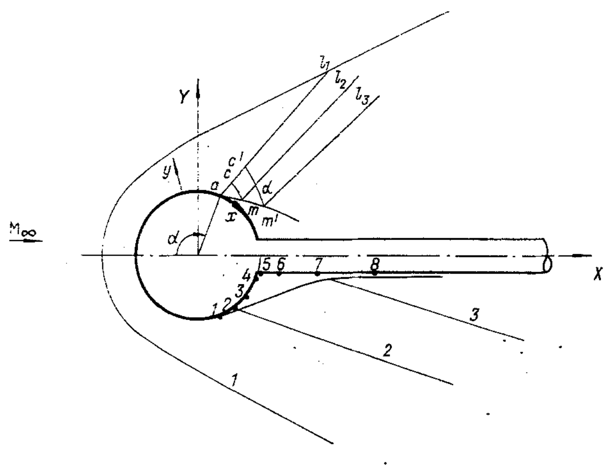

Fig. 4

this purpose, we use the parameters F_3, F'_3, and h^*, applying Eq. (1.2) and the dependence $H^* = H^*(H)$ used in [5] in order to calculate h^*. Once we obtain D(x), we repeat the integration of the first two equations of Eq. (1.1) until the values δ^*/δ_1^* and $\delta^{**}/\delta_1^{**}$ of the last approximation no longer differ from the preceding values by 10^{-5}.

The dependences $H = H(\Lambda)$ thus calculated in the separation zone of a boundary layer when M_1 is between 1.56 and 3.0 and Re_{δ_1} is between $3 \cdot 10^4$ and $11 \cdot 10^4$ are depicted in Fig. 2. Here 1 represents $M_1 = 2.92$; 2, $M_1 = 3.00$; 3, $M_1 = 2.49$; 4, $M_1 = 1.56$; 5, $M_1 = 2.32$; 6, $M_1 = 1.79$; and 7, $M_1 = 2.4$. The resulting data are approximated by a theoretical dependence $H = H(\Lambda)$ calculated for an equilibrium incompressible turbulent boundary layer [5, 8] in the region of the attached boundary layer. The results can be approximated by a linear function having the value $H = H_3 = 10$ when $\Lambda = 0$ in the zone of circulatory flow.

§2. We may calculate the pressure distribution in the separation region by integrating the system of equations (1.1) after we obtain the dependence $H = H(\Lambda)$ throughout the entire interaction region. The equations $\Gamma = \Gamma(H)$, $A^* = A^*(M^*)$, $H^* = H^*(H)$, and $M^* = 0.5(M_1 + M_2)$ given in [5], as well as the dependence $H = H(\Lambda)$ we have found (cf. Fig. 2), are used in the calculations. The parameter β is calculated for two-dimensional flow from the Prandtl–Meyer equation and by the method of characteristics for axisymmetrical flow. Since the system of equations (1.1) is a system of ordinary differential equations, it is sufficient to indicate the set of parameters M_1, δ_1^*, δ_1^{**}, and β_1 in the initial interaction section 1. If the position of section 1 is known (for example, from experiment), we may calculate the critical pressure ratio $p_3^0 = p_3/p_1$ at which the turbulent boundary layer separates. For this purpose, we will carry out the calculation from section 1 (the parameter $H \approx 1.3$) to section 3. The calculation is assumed valid when H reaches the value $H = H_3 = 10.0$, which is the finite boundary condition in section 3.

On the other hand, whenever the position of the interaction starting section is unknown, it is necessary to impose additional conditions on the system of equations (1.1), i.e., to specify terminal boundary conditions. It is convenient to select full conditions on a two-dimensional plate (specifying $\Lambda_8 = 0$ and $\beta_8 = 0$) as the additional conditions in the attached region, when calculating flow in the neighborhood of a flap inducing separation of the boundary layer (cf. Fig. 1). Calculations of two-dimensional flow in the neighborhood of a concave corner (flap) will first be carried out in the separation zone and then in the attached zone. The values of the boundary-layer parameters in section 1 will be calculated in accordance with the position we have supposed. The position of the interaction starting section is refined in the course of the solution. The calculation is carried out from section 1 to section 3, resulting in δ_3^*, δ_3^{**}, M_3 and β_3 in the separation zone. We calculate the length of the constant-pressure flow zone 3-6. The calculation is carried out using a previous [9] method under the assumption that flow into the attached zone is equivalent to flow in the base region behind a step of depth $b = (x_4 - x_3) \sin \alpha^*$ (cf. Fig. 1); the constant-pressure flow parameters of the base region M, β, and δ^{**} are as follows, taking into account the wedge angle α^*: $M = M_6 = M_3$, $\beta = \beta_6 = \beta_3 - \alpha^*$, and $\delta^{**} = \delta_6^{**} = \delta_3^{**}$. The displacement thickness has the form

$$\delta_6^* = (\delta_3^* + b) + [(x_6 - x_5) + (x_4 - x_3)\cos\alpha^*] \operatorname{tg}\beta_6.$$

We then consider the attached boundary-layer region 6-8, which is calculated using a previous [5] method. The flow conditions on the wedge surface are selected in the same way as for a two-dimensional

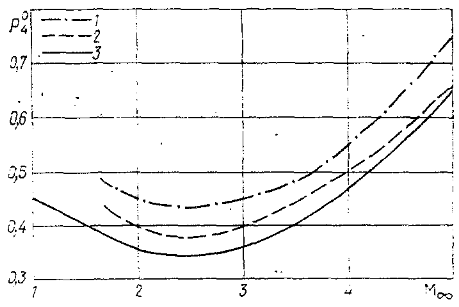

Fig. 5

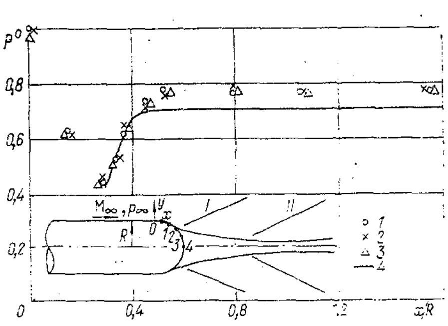

Fig. 6

plate, i.e., $\beta_8 = 0$ and $\Lambda_8 = 0$, at the end of the interaction region. The boundary conditions at the end of the interaction zone for a given wedge generatrix angle α^* are satisfied by selecting the position of the starting section 1 by the range method. Our calculations demonstrated that the length $(x_6 - x_5)$ of the constant-pressure zone 5-6 cannot be computed, and must be set equal to zero when calculating the attached zone for small separation regions in which no constant-pressure zone 3-4 occurs (in this case, it has been shown by Bonderav [3], who processed his experimental data in separation zones in front of a flap, that $\alpha^* \leq 2\beta_3$).

Calculated pressure distributions $p^0 = p/p_1$ in the interaction zone for $M_1 = 2.0$, 2.7, and 3.0 are compared in Fig. 3 to previous [10] experimental data. Curves 1-3 correspond to $M_1 = 2.0$, $\alpha^* = 20°$; $M = 2.7$, $\alpha^* = 25°$; and $M = 3.0$, $\alpha^* = 25°$; $L = 140$ mm. We should bear in mind that experimental data in the attached boundary-layer region on the flap are available for the entire interaction zone only for the case $M_1 = 2.7$. Therefore, the calculation can be compared to experiment when $M_1 = 2.0$ and 3.0 only for the separation zone.

§3. Let us consider flow arising near the surface of a spherical body streamlined by supersonic flow (Fig. 4). A departed shock wave 1 is observed in front of the body. The parameters of the undisturbed gas in front of the shock wave are M_∞, p_∞, and ρ_∞. Gas velocity is equal to zero at the leading critical point on the surface of the body while pressure and density p'_0 and ρ'_0 are, correspondingly, calculated from the parameters of stagnant flow behind the plane shock wave. The x-coordinate is counted off along the generatrix of the surface of the body and is determined by the central angle $\alpha = x/R$, where R is the radius of the sphere.

The appearance of a shock wave 2 at the separation point of a boundary layer from the surface of the body is the distinctive feature in the formation of the bottom wake of a spherical body [4]. The flow assumed the direction of the drag axis passing through the shock wave 3 in the attached boundary-layer region. The total flow on the body may be divided into three zones. The first zone is from the leading critical point to section 1, the second is the separation zone 1-4, and the third is the attached boundary-layer zone 5-8. Well-known experimental and theoretical studies [11] of sphere streamline in an attached flow region (in the first zone) give us grounds for asserting that the dimensionless pressure p/p'_0 monotonically decreases as α increases in the range of angles from 0 to α_1. Experiments carried out in a separation zone on a spherical intake part of a body demonstrate that pressure increases with increasing α in the separation zone 1-3, reaching some constant value in the zone 3-4 (base pressure value).

We will use the system of equations (1.1) to calculate flow in a separation zone. The calculation will be carried out from section 1 (cf. Fig. 4) downstream the main flow, to section 3. Here we will use the equations given in Sec. 1 between the parameters H, H*, Λ, Γ, A*, and M*. Results of a numerical solution obtained [11] for ideal gas flows were used to calculate the flow parameters in section 1. It should be noted that the boundary-layer equations (1.1) were written in an orthogonal x, y-coordinate system, the x-coordinate being counted off along the body generatrix. Equations for an external ideal supersonic flow derived by the method of characteristics can be conveniently written in an orthogonal XY-coordinate system, where the X axis is directed along the axis of symmetry of axisymmetrical flow. It is therefore also necessary to take into account in the calculation equations relating both coordinate planes. Here

$$\beta = \beta^* - \beta_*,$$

where β^* is the angle between the direction of the velocity vector on the boundary of the boundary layer and the X axis and β_* is the angle between the x and X axes.

Flow on the intake part of a sphere with a cylindrical base support resembles flow in the neighborhood of an "axisymmetrical flap." The calculation of the interaction of a turbulent boundary layer with an external ideal supersonic flow is analogous in this case to the calculation suggested in Sec. 2.

Let us calculate flow in the neighborhood of a spherical intake part of a body with base bracket. We first calculate the flow in the separation zone 1-3. After the parameters in section 3 have been determined, we then calculate the length of the constant-pressure flow zone 3-6. It is calculated using a previous [9] method under the assumption that flow in the base region with step of depth $b = R \cos \alpha_3 - r_2$ occurs in this zone (cf. Fig. 4), the constant-pressure flow parameters in the base region (M and δ^{**}) having the form

$$M = M_6 = M_3; \quad \delta^{**} = \delta_6^{**} = (\delta_3^{**} R \cos \alpha_3) / r_2,$$

and the displacement thickness given by

$$\delta_6^* \approx (\delta_3^* + b) + \int_{x_3}^{x_6} \operatorname{tg} \beta \, dx.$$

The attached boundary-layer region 6-8 is calculated using a previous [5] method. Calculation of the external ideal flow (to obtain the parameter β^*) is carried out by the method of characteristics.

Let us write equations for the characteristics of the first and second families in a physical XY plane and in the motion hodograph plane [13],

$$\frac{dY}{dX} = \frac{\mu A \pm B}{\mu B \mp A}; \quad Edl \pm Ddz \pm LdX = 0. \tag{3.1}$$

We will have on the streamlines,

$$\frac{dY}{dX} = \frac{A}{B}; \quad \frac{dw_*^2}{2} + Fdz = 0; \quad dt = \frac{dz}{\varkappa}, \tag{3.2}$$

where

$$A = l(1 - l^2); \quad B = \frac{1}{4}(1 - l^2)^2 - l^2; \quad E = \frac{4}{1 + l^2}; \quad D = \frac{\mu}{\varkappa(\mu^2 + 1)};$$

$$L = \frac{A}{(\mu B \mp A) Y}.$$

The F. É. Élers variables $\mu = \sqrt{M^2 - 1}$, $l = \operatorname{tg}(\beta^*/4)$, $z = \ln(p/p_1)$, $t = \ln(\rho/\rho_1)$, and $w_* = w\sqrt{p_1/\rho_1}$ were taken as the unknown functions.

We calculate the flow on an internal streamline $a$mm' (cf. Fig. 4) using the characteristics of the second family (3.1) and the streamline equation (3.2). We obtain as the final differences

$$Y_m - Y_a = (X_m - X_a)(A/B)_a, \; (Y_m - Y_c) = (X_m - X_c)[(\mu A - B)/(\mu B + A)]_c.$$

In the motion hodograph plane we have for the characteristics of the second family

$$l_m = l_c + \frac{D_c}{E_c}(z_m - z_c) + \frac{L_c}{E_c}(X_m - X_c).$$

Equations (1.1) were used to calculate the z_m values. We compute the characteristic c'd m', the streamline element mm', and so on after we have obtained the characteristic cm. Thus, we find the solution for external ideal supersonic flow.

A calculation using the F. É. Élers variables significantly shortens computation time in comparison with the ordinary calculation using the variables $\mu = \arctan\sqrt{M^2 - 1}$ and β^*. The conditions $\beta_8 = 0$ and $\Lambda_8 = 0$ in section 8 are used for the system of equations (1.1) as the terminal boundary conditions. Our method was used for a base support of radius $r_2 \gg \delta_1$. Figure 5 depicts the dependence of base pressure $p_4^0 = p_4/p_\infty$ behind spherical bodies on the number M_∞ calculated as M_∞ varied from 1.7 to 5.0 and when $r_2/R = 0.25$. The function $p_4^0 = f(M_\infty)$ has a minimum at $M_\infty \approx 2.5$. Curve 1 corresponds to $\delta_1^{**}/R = 0.005$, while curve 2, to $\delta_1^{**}/R = 0.003$, and curve 3, to results of a calculation ($\delta_1^{**}/R = 0$) carried out using a previous [4] method.

§4. An experimental study of the interaction of a turbulent boundary layer with an external supersonic flow was carried out for the spherical intake part of a cylindrical body. We used an annular contoured nozzle with center body (the model itself was used as the center body) designed for $M_\infty = 1.9$. The model diameter $d = 2R = 20$ mm; static pressure was selected by means of tubes with internal diameter $d_1 = 0.5$ mm.

Figure 6 depicts the flow scheme in the base region of a cylindrical body with spherical intake part. A pressure shock I appears in the region of separation of the boundary layer, and flow assumed the direction of the drag axis passing through the pressure shock II in the attached flow region. Sections denoted by the digits 0–3 correspond to flow spreading onto the spherical intake part of the body (0), start of interaction in the separation zone (1), separation of boundary layer (2), and pressure leveling (3). Both a gradient (1–3) and a gradientless flow zone (3–4) can be distinguished in the separation zone 1–4.

The Reynolds number, calculated using the parameters of the incoming flow to the edge of the boundary layer and the length of the center body from the nozzle "jet," varied in the range from $4.1 \cdot 10^6$ to $9.6 \cdot 10^6$. In Fig. 6, 1 corresponds to $Re = 4.1 \cdot 10^6$, 2, to Re $6.8 \cdot 10^6$, and 3, to $Re = 9.6 \cdot 10^6$. Measurements of the total pressure profiles carried out using a total pressure pipe (micro-Pitot) in the section directly in front of the spherical part of the body when $Re = 6.4 \cdot 10^6$ demonstrated that n was approximately between 1/7 and 1/8 and that the momentum thickness $\delta_0^{**}/d = 0.0035$ after scaling to the velocity profile $u/u_1 = (y/\delta)^n$.

Results of measurements of the pressure distribution $p^0 = p/p_\infty$ obtained on the spherical intake part of a body are presented in Fig. 6.

A calculation was carried out using the theory developed above by assuming that the base pressure behind the spherical intake part with base support $r_2/R = 0.25$ was practically equal to the base pressure without the support (which was experimentally proved to within 3–10% for cylindrical bodies and cones with a two-dimensional end intake part). Results of the calculation are depicted in Fig. 6 (curve 4).

In conclusion, the author wishes to express his appreciation to M. Ya. Yudelovich and E. N. Bondarev for useful remarks and discussion of the study.

NOTATION

x, y, longitudinal and transverse coordinates; δ, δ^*, δ^{**}, boundary-layer thickness, displacement thickness, and momentum thickness; θ, θ^*, θ^{**}, layer thickness, displacement thickness, and momentum thickness of an incompressible boundary layer; u, ρ, longitudinal velocity and density of a compressible boundary layer; U, ρ', longitudinal velocity and density of an incompressible boundary layer; Λ, pressure gradient parameter of an incompressible boundary layer; w, velocity; M, p, Mach number and pressure; a, speed of sound; r, radius; I, enthalpy; α^*, angle of inclination of the flap; τ, frictional stress; ν, Prandtl–Meyer angle; Pr, Prandtl number; r_1, radius of the base part of the axisymmetrical part of the body; r_2, radius of the base support; b, depth of step; $\varepsilon = 0$ for two-dimensional flow; β, angle between the direction of velocity of the external edge of the boundary layer and the surface of the body; $\varepsilon = 1$ for axisymmetrical flow. Indices: 0, stagnant flow; 1, on the external edge of the boundary layer or at the start of the zone within which the boundary layer interacts with an external ideal flow; w, parameters on the wall; ', for incompressible flow.

$$h = \frac{\delta^*}{\delta^{**}};\ h^* = \frac{\delta^*}{\delta};\ h^{**} = \frac{\delta^{**}}{\delta};\ H = \frac{\theta^*}{\theta^{**}};\ H^* = \frac{\theta^*}{\theta};\ H^{**} = \frac{\theta^{**}}{\theta};$$

$$\tau_w = \gamma^2 \rho_1 u_1^2;\ \tau_\omega' = \Gamma^2 \rho_1' U_1^2;\ \varkappa = c_p/c_V;\ \vartheta(\varkappa + 1)/(\varkappa - 1);$$

$$\nu = \vartheta^{0.5} \operatorname{arctg}[(M^2 - 1)/\vartheta]^{0.5} - \operatorname{arctg}(M^2 - 1)^{0.5};$$

$$\lambda^2 = \left|\frac{\delta^*}{u_1} \frac{du_1}{dx}\right|.$$

LITERATURE CITED

1. H. McDonald, "Study of turbulent separated flow arising as a concave corner is streamlined by supersonic flow," in: Mekhanika [Periodic collection of translations of foreign articles], No. 1 (95) (1966).
2. L. V. Gogish, "Approximate calculation of critical attachment and separation pressures of a turbulent boundary layer in supersonic flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1968).
3. A. N. Antonov, and E. N. Bondarev, "Approximate method of calculating a turbulent boundary layer with positive pressure gradient," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1968).
4. M. Ya. Yudelovich, "Approximate technique of calculating base pressure for spherical bodies," Izv. Akad. Nauk SSSR, Mekhanika, No. 3 (1965).

5. A. N. Antonov, "Calculation of the interaction of a turbulent boundary layer with an external supersonic flow behind a step," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1971).
6. D. M. Kuehn, "Turbulent boundary-layer separation induced by flares on cylinders at zero angle of attack," NASA Technical Report R-117 (1961).
7. Bogdonov and Kepler, "Separation of a turbulent boundary layer in supersonic flow," in: Voprosy Raketn. Tekh. [Periodic collection of translations of foreign articles], No. 6 (1956).
8. J. L. Mellor and D. M. Gibson, "Equilibrium turbulent boundary layers," in: Mekhanika [Periodic collection of translations of foreign articles], No. 2 (102) (1967).
9. H. McDonald, "The turbulent supersonic base pressure problems in comparison between a theory and some experimental evidence," Aeronaut. Quart., No. 17 (1966).
10. D. Chapman, D. Kuehn, and H. Larson, "Investigation of separated flows in supersonic and subsonic streams with an emphasis on the transition," NASA Report No. 1356 (1956).
11. Yu. N. D'yakonov and N. A. Zaitsev, "Streamline of a blunt body by supersonic flow of an ideal gas," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mashinostr., No. 1 (1963).

INTERACTION OF AN EXTERNAL DISTURBANCE WITH TURBULENT FLOW

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Previous calculations [1] and a critical analysis of the interpretation of some experimental data [2, 3] are verified and refined. A model is proposed that directly takes into account in the motion equations terms describing the interaction of the disturbance with turbulent oscillations. The advantages of such an approach in comparison with the use of turbulent viscosity models are demonstrated.

Interest in the stability of turbulent flows has recently grown in connection with attempts to predict the averaged characteristics of turbulent flow based on stability properties [4-7]. The stability problem as of now has been solved only in a quasilaminar approximation, in which the interaction of the disturbance with fluctuations is not taken into account [5]. This is due to the absence of experimental data that would permit any given model describing such interaction to be accepted. A series of works by Reynolds and Hussain [1-3], in which original experiments and the first calculations using models taking into account the interaction of a weak nonrandom signal from the turbulence for channel flow were performed, appeared in 1970-1972.

A periodic perturbation (vibrating streaks near walls) was introduced in a given section of the channel and its downstream propagation was studied. A weak, nonrandom signal consisting of about 4% of the turbulent velocity fluctuations was isolated. Experiments were carried out for four frequencies with a Reynolds number (Re = 13,800) calculated according to the channel half-width and maximal velocity [2].

A spatial stability problem for turbulent flow to a linear approximation arose as a result of this experiment. The exponential nature of signal attenuation was indicated by the validity of the linear approximation [2-3].

The disturbance equations have the form

$$\frac{\partial \langle v_i \rangle}{\partial x_j} + \frac{\partial (U_j \langle v_i \rangle + U_i \langle v_j \rangle)}{\partial x_j} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial^2 \langle v_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle v_i v_j^m + v_i^m v_j \rangle; \quad \frac{\partial \langle v_j \rangle}{\partial x_j} = 0, \tag{1}$$

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